10. Hausübung, **Statistische Physik**

abzugeben am Donnerstag, 5.1.2012

Aufgabe H19 *Van der Waals gas* (5 Punkte)

Consider the van der Waals gas with fixed number N of particles.

a. Express the entropy σ of the van der Waals gas as function of the volume V and the temperature τ. (You can use the quantum density shorthand $n_Q(\tau)$ for tidiness.)

b. Express the total energy U similarly.

c. Express the enthalpy $H = U + pV$ both as function of τ and V and as function of τ and the pressure p, but only to first order in the parameters a and b.

Aufgabe H20 *First-order crystal transformation* (7 Punkte)

Consider a crystal that can exist in either of two structures, denoted by α and β. We suppose that the α structure is the stable low temperature form and the β structure is the stable high temperature form of the substance. If the zero of the energy scale is taken as the state of separated atoms at infinity, then the energy *density* $U(0)$ at $\tau = 0$ will be negative. The phase stable at $\tau = 0$ will have the lower value of $U(0)$: thus $U_\alpha(0) < U_\beta(0)$. If the velocity of sound v_β in the β phase is lower than v_α in the α phase, corresponding to lower values of the elastic moduli for β, then the thermal excitations in the β phase will have larger amplitudes than in the α phase. The larger the thermal excitations, the larger the entropy and the lower the free energy. Soft systems tend to be stable at high temperatures, hard systems at low.

a. Show that the free energy density contributed by the phonons in a solid at a temperature much less than the Debye temperature is given by $-\pi^2\tau^4/30v^3\hbar^3$, in the Debye approximation with v taken as the velocity of all phonons.

b. Show that the transformation temperature τ_c between the two phases satisfies

$$\tau_c^4 = (30\hbar^3/\pi^2)\frac{U_\beta(0) - U_\alpha(0)}{v_\beta^{-3} - v_\alpha^{-3}}.$$

There will be a finite real solution if $v_\beta < v_\alpha$. This example is a simplified model of a class of actual phase transformations in solids.

c. The latent heat of a transformation is defined as the thermal energy that must be supplied to carry the system through the transformation. Show that the latent heat for this model is

$$L = 4[U_\beta(0) - U_\alpha(0)].$$

Hint: neglect any change of volume per particle between the two phases.